Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 24, 2003*
Accepted: *January 21, 2004*

ε'/ε at the NLO: 10 years later

Andrzej J. Buras
Physik Department, Technische Universität München,
D-85748 Garching, Germany
E-mail: `Andrzej.Buras@ph.tum.de`

Matthias Jamin
Institut für Theoretische Physik, Universität Heidelberg,
Philosophenweg 16, D-69120 Heidelberg, Germany
E-mail: `Jamin@Uni-HD.DE`

Abstract: During the last four years several parameters relevant for the analysis of the CP-violating ratio ε'/ε improved and/or changed significantly. In particular, the experimental value of ε'/ε and the strange quark mass decreased, the uncertainty in the CKM factor has been reduced, and for a value of the hadronic matrix element of the dominant electroweak penguin operator Q_8, some consensus has been reached among several theory groups. In view of this situation, ten years after the first analyses of ε'/ε at the next-to-leading order, we reconsider the analysis of ε'/ε within the SM and investigate what can be said about the hadronic Q_6 matrix element of the dominant QCD penguin operator on the basis of the present experimental value for ε'/ε and todays values of all other parameters. Employing a conservative range for the reduced electroweak penguin matrix element $R_8 = 1.0 \pm 0.2$ from lattice QCD, and present values for all other input parameters, on the basis of the current world average for ε'/ε, we obtain the reduced hadronic matrix element of the dominant QCD penguin operator $R_6 = 1.23 \pm 0.16$ implying $\langle Q_6\rangle_0^{\rm NDR}(m_c) \approx -0.8\,\langle Q_8\rangle_2^{\rm NDR}(m_c)$. We compare these results with those obtained in large-N_c approaches in which generally $R_6 \approx R_8$ and $\langle Q_6\rangle_0^{\rm NDR}(m_c)$ is chirally suppressed relatively to $\langle Q_8\rangle_2^{\rm NDR}(m_c)$. We present the correlation between R_6 and R_8 that is implied by the data on ε'/ε provided new physics contributions to ε'/ε can be neglected.

Keywords: Standard Model, Kaon Physics, CP violation.

Contents

1. Introduction

The ratio ε'/ε that parametrises the size of direct CP violation with respect to the indirect CP violation in $K_L \to \pi\pi$ decays has been the subject of very intensive experimental and theoretical studies in the last three decades. After tremendous efforts, on the experimental side the world average based on the recent results from NA48 [1, 2] and KTeV [3, 4], and previous results from NA31 [5, 6] and E731 [7], reads

$$\frac{\varepsilon'}{\varepsilon} = (16.6 \pm 1.6) \cdot 10^{-4} \qquad (2003)\,. \tag{1.1}$$

A recent discussion of the experimental issues involved in these analyses can be found in ref. [8].

On the other hand, the theoretical estimates of this ratio are subject to very large hadronic uncertainties. While several analyses of recent years within the Standard Model (SM) find results that are compatible with (1.1) [9]–[18], it is fair to say that even ten years after the first analyses of ε'/ε at the next-to-leading order (NLO) [19, 20], the chapter on the theoretical calculations of ε'/ε is certainly still open. A full historical account of the theoretical efforts before 1998 can for example be found in the reviews [21, 22].

It should be emphasised that all existing analyses of ε'/ε use the NLO Wilson coefficients calculated by the Munich and Rome groups in 1993 [19, 20, 23, 24, 25, 26], but the hadronic matrix elements, the main theoretical uncertainty in ε'/ε, vary from paper to paper. Nevertheless, apart from the hadronic matrix element of the dominant QCD penguin operator Q_6, in the last years progress has been made with the determination of all other relevant parameters, which enter the theoretical prediction of ε'/ε. In view of this situation, we think it is legitimate to reconsider the analysis of ε'/ε within the SM and to investigate what can be said about the hadronic Q_6 matrix element on the basis of the present experimental value of ε'/ε and todays values of all other parameters. In doing this we assume that any potential new physics contributions to ε'/ε can be neglected. We will return to this point at the end of our paper.

The phenomenological approach towards the values of the hadronic matrix elements presented in this work is a continuation of our approach in [19], where we extracted the matrix elements of current-current operators Q_i $(i = 1 - 4)$ from the CP conserving data. In that case the assumption about the absence of significant new physics contributions to the relevant amplitudes was not necessary. On the contrary, the present analysis has this particular limitation. Nevertheless, as we shall see below some insight into the pattern of the size of the hadronic matrix elements of penguin operators can be gained.

In order to set the scene for our investigation let us recall that after NA31 in 1993 [6] and KTeV in 1999 [4] presented surprisingly high values for ε'/ε, the 1999 world average including the E731 value [7] was

$$\frac{\varepsilon'}{\varepsilon} = (21.8 \pm 3.0) \cdot 10^{-4} \qquad (1999)\,, \tag{1.2}$$

being roughly 30% higher than the 2003 value. Within our 1999 analysis of ε'/ε [17] that was performed using matrix elements of penguin operators Q_6 and Q_8 in the ballpark of the values obtained in the large-N_c approach of [27, 28, 29], it was essentially impossible to reproduce the value in (1.2). We found ε'/ε typically a factor of two to three below the 1999 experimental value.

As we shall see below, the confrontation of the SM with the 2003 experimental value for ε'/ε as given in (1.1), does not resemble the 1999 situation presented by us in [17]. The following important facts are responsible for this change:

- The experimental value of ε'/ε decreased approximately by a factor of 1.3.
- The value of m_s, relevant for ε'/ε within the approach of [27, 28, 29], decreased by roughly 15%, enhancing the theoretical value of ε'/ε by roughly a factor of 1.3.
- The isospin breaking parameter $\Omega_{\rm IB}$ decreased from roughly 0.25 to 0.06, enhancing ε'/ε by another factor of 1.4.
- The value for the hadronic matrix element of the dominant electroweak penguin operator $\langle Q_8\rangle$ became more robust due to general consistency of results from lattice QCD and other analytic non-perturbative approaches to be discussed below.
- Finally, the value of the CKM factor $\mathrm{Im}\lambda_t$ became more precise, although it did not change significantly compared to our 1999 value.

As a result of all these changes, our SM value for ε'/ε presented below turns out to be fully consistent with the experimental value given in (1.1) for values of the hadronic matrix elements $\langle Q_6\rangle$ and $\langle Q_8\rangle$ which do not differ by more than 20% from the ones obtained within the large-N_c approach of [27, 28, 29]. On the other hand, the chiral suppression of $\langle Q_6\rangle_0$ with respect to $\langle Q_8\rangle_2$ that is characteristic for the latter approach, is only partially supported by the data.

In the next section, we recollect the basic equations which are required for the analysis of ε'/ε within the SM. In section 3, we present the details and numerical results of this work, in section 4 we compare our results with those obtained in various approaches, and in section 5 we summarise and draw some conclusions.

2. Basic formulae

The central formula for ε'/ε of refs. [17, 30, 19] takes the following form:

$$\frac{\varepsilon'}{\varepsilon} = \mathrm{Im}\lambda_t \cdot F_{\varepsilon'}(x_t) \tag{2.1}$$

where

$$\lambda_t = V_{ts}^* V_{td}\,, \qquad x_t = \frac{m_t^2}{M_W^2} \tag{2.2}$$

and

$$F_{\varepsilon'}(x_t) = P_0 + P_X\, X_0(x_t) + P_Y\, Y_0(x_t) + P_Z\, Z_0(x_t) + P_E\, E_0(x_t)\,. \tag{2.3}$$

The functions X_0, Y_0, Z_0 and E_0 are given by ($m_t = 165\,\mathrm{GeV}$)

$$X_0(x_t) = \frac{x_t}{8}\left[\frac{x_t+2}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2}\,\ln x_t\right] = 1.505\,, \tag{2.4}$$

$$Y_0(x_t) = \frac{x_t}{8}\left[\frac{x_t-4}{x_t-1} + \frac{3x_t}{(x_t-1)^2}\ln x_t\right] = 0.962\,, \tag{2.5}$$

$$\begin{aligned} Z_0(x_t) &= -\frac{1}{9}\ln x_t + \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t-1)^3} + \\ &\quad + \frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t-1)^4}\ln x_t = 0.664 \end{aligned} \tag{2.6}$$

$$E_0(x_t) = -\frac{2}{3}\ln x_t + \frac{x_t^2(15-16x_t+4x_t^2)}{6(1-x_t)^4}\ln x_t + \frac{x_t(18-11x_t-x_t^2)}{12(1-x_t)^3} = 0.271\,. \tag{2.7}$$

The coefficients P_i are given in terms of the non-perturbative parameters R_6 and R_8 as follows:

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8\,, \tag{2.8}$$

where the coefficients $r_i^{(0)}$, $r_i^{(6)}$ and $r_i^{(8)}$ comprise information on the Wilson-coefficient functions of the $\Delta S = 1$ weak effective hamiltonian at the next-to-leading order [23, 24, 25, 26], and their numerical values for different values of $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$ at $\mu = m_c$ in the NDR renormalisation scheme are displayed in table 1. This table updates the corresponding ones presented in [17, 30].

In terms of the hadronic B-parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$, as well as the strange quark mass, the non-perturbative parameters R_6 and R_8 are defined as

$$R_6 \equiv B_6^{(1/2)}\left[\frac{121\,\mathrm{MeV}}{m_s(m_c)+m_d(m_c)}\right]^2, \qquad R_8 \equiv B_8^{(3/2)}\left[\frac{121\,\mathrm{MeV}}{m_s(m_c)+m_d(m_c)}\right]^2. \tag{2.9}$$

In the large-N_c approach of [27, 28, 29], the matrix elements of the dominant QCD-penguin

	$\Lambda^{(4)}_{\overline{\rm MS}} = 310\,{\rm MeV}$			$\Lambda^{(4)}_{\overline{\rm MS}} = 340\,{\rm MeV}$			$\Lambda^{(4)}_{\overline{\rm MS}} = 370\,{\rm MeV}$		
i	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$
0	-3.574	16.552	1.805	-3.602	17.887	1.677	-3.629	19.346	1.538
X_0	0.574	0.030	0	0.564	0.033	0	0.554	0.036	0
Y_0	0.403	0.119	0	0.392	0.127	0	0.382	0.134	0
Z_0	0.714	-0.023	-12.510	0.766	-0.024	-13.158	0.822	-0.026	-13.855
E_0	0.213	-1.909	0.550	0.202	-2.017	0.589	0.190	-2.131	0.631

Table 1: The coefficients $r_i^{(0)}$, $r_i^{(6)}$ and $r_i^{(8)}$ of formula (2.8) for various $\Lambda^{(4)}_{\overline{\rm MS}}$ in the NDR scheme.

(Q_6) and the dominant electroweak penguin (Q_8) operator, are then given by [31, 19]

$$\langle Q_6\rangle_0 \;=\; -4\sqrt{\frac{3}{2}}\,(F_K-F_\pi)\left(\frac{m_K^2}{121\,\text{MeV}}\right)^2 R_6 = -0.597\cdot R_6\,\text{GeV}^3\,, \tag{2.10}$$

$$\langle Q_8\rangle_2 \;=\; \sqrt{3}\,F_\pi\left(\frac{m_K^2}{121\,\text{MeV}}\right)^2 R_8 = 0.948\cdot R_8\,\text{GeV}^3\,, \tag{2.11}$$

where for $\langle Q_8\rangle_2$, we have neglected a small 1% correction which is independent of m_s, and the numerical values have been obtained by employing values for F_π, F_K and m_K as given in the current PDG [32]. Let us remark that the overall normalisation of our matrix elements in (2.10) and (2.11) differs by a factor $\sqrt{3/2}$ from another convention used in the literature. This factor, however, compensates with a corresponding factor in our definition of the isospin amplitudes to provide the same physical results.

In the strict large-N_c limit, $B_6^{(1/2)} = B_8^{(3/2)} = 1$, the μ-dependence of the matrix elements $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ is fully governed by the μ-dependence of the quark masses $m_{s,d}(\mu)$, and, as the mixing with other operators can be neglected in this limit, the evolution of the Wilson coefficients $C_6(\mu)$ and $C_8(\mu)$ precisely cancels the μ-dependence of $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ such that ε'/ε is μ-independent as it should be. As our numerical analysis of ref. [19] demonstrated, $B_6^{(1/2)}$ and $B_8^{(3/2)}$ remain independent of μ to a very good accuracy also outside the strict large-N_c limit, provided these parameters do not deviate considerably from unity.

The formulae (2.9), together with the strict large-N_c limit, also imply $R_6 = R_8$, so that in this limit there is a one-to-one correspondence between ε'/ε and $R_6 = R_8$ for fixed values of the remaining parameters. Consequently, in this case, only for certain values of $m_s(m_c)$ is one able to obtain the experimental value for ε'/ε. This has been emphasised in particular in [33], and we shall return to this point in the next section. Thus in this approach a concrete prediction for ε'/ε can be made provided the value $m_s(m_c)$ has been determined by some other method. Moreover, once $m_s(m_c)$ is known, also R_6, R_8, $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ are known, but they always satisfy the relations:

$$\frac{R_6}{R_8} = 1\,, \qquad \frac{\langle Q_6\rangle_0}{\langle Q_8\rangle_2} = -\,0.63\,. \tag{2.12}$$

We shall compare the relations (2.12) with our results for $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ obtained by various methods below.

JHEP01(2004)048

3. Numerical analysis

3.1 Preliminaries

Before we come to a detailed presentation of our numerical analysis, let us first briefly discuss the origin of the values for the dominant input parameters that we have used in our analysis.

The dependence of ε'/ε on the quark mixing or CKM matrix elements appears through the parameter $\mathrm{Im}\lambda_t$. Fits determining the elements of the CKM matrix have been the subject of extensive efforts in the last years. The present status has been summarised very recently in the review [34]. Employing the best fit incorporating all known constraints within the SM, the resulting value for $\mathrm{Im}\lambda_t$ is then found to be [35]:

$$\mathrm{Im}\lambda_t = (1.31 \pm 0.10) \cdot 10^{-4} . \tag{3.1}$$

The next parameter which deserves an explicit mentioning since ε'/ε is rather sensitive to its precise value, is the quantity $\Omega_{\rm IB}$, parametrising isospin-breaking corrections which result from strong isospin violation ($m_u \neq m_d$) and electromagnetic corrections. For this parameter, we employ the result of the very recent analysis [36] (see also refs. [37, 38]): $\Omega_{\rm IB} = 0.06 \pm 0.08$. The shift from $\Omega_{\rm IB} = 0.25$, that was used in our previous analysis of ε'/ε [17], to the new result, has the largest impact for the theoretical value of ε'/ε.

An important fundamental QCD input parameter is the value of the strong coupling constant α_s. As pointed out in refs. [39, 17], ε'/ε is roughly proportional to the QCD parameter $\Lambda_{\overline{\rm MS}}$ in a four-flavour theory, when this parameter is $\mathcal{O}(350\,\mathrm{MeV})$. For the strong coupling, we have used the recent average $\alpha_s(M_Z) = 0.119 \pm 0.002$, corresponding to $\Lambda^{(4)}_{\overline{\rm MS}} = 340 \pm 30\,\mathrm{MeV}$.

Another parameter is the top quark mass that enters the Inami-Lim functions given above and the fit of $\mathrm{Im}\lambda_t$. However, m_t is already well known and its precise value is less important. On the other hand, the value of the strange quark mass m_s plays an important role in the large-N_c approach discussed in the previous section. For the quark masses, we shall then use the following values:

$$m_t(m_t) = 165 \pm 5\,\mathrm{GeV}\,, \qquad m_s(m_c) = 115 \pm 20\,\mathrm{MeV}\,. \tag{3.2}$$

The top mass evaluated at the scale m_t results from converting the corresponding pole mass value [32], and the strange quark mass is an average over recent determinations of this quantity [40, 41, 42, 43, 44]. Due to our method of extracting the matrix elements of current-current operators Q_i ($i = 1-4$) from the CP conserving data [19], the strange mass is required at the scale $m_c = 1.3\,\mathrm{GeV}$ and our central value would correspond to $m_s(2\,\mathrm{GeV}) = 100\,\mathrm{MeV}$.

As stressed by several authors in the literature, the strong m_s dependence of $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ given in the large-N_c approach is somewhat artificial. In fact, the lattice studies [45] and the discussion in refs. [13, 46, 9] indicate that these matrix elements are only weakly dependent on the actual value of m_s since they are related to the quark condensate via the

JHEP01(2004)048

GMOR relation [47, 48]. Here we shall follow first a different approach. We will directly deal with the parameters R_6 and R_8 of eq. (2.9), and only at the end of our numerical analysis, we shall discuss the implications for $B_6^{(1/2)}$ and $B_8^{(3/2)}$.

3.2 Present knowledge of R_8

As can be inferred from the formulae presented in the last section, in order to be able to calculate ε'/ε, we still require some input for R_6 and R_8 which parametrise the dominant contributions from QCD and electroweak penguin operators respectively. Concerning the parameter R_8, in the last years progress has been achieved both in the framework of lattice QCD [49, 50, 51] as well as with analytic methods to tackle QCD non-perturbatively [52, 53, 54, 55, 9].

The current situation of the status of $\langle Q_8\rangle_2$ calculations has been summarised nicely in the recent article [55]. At present, the most precise determination of $\langle Q_8\rangle_2$ comes from the lattice QCD measurement [51], corresponding to $R_8 = 1.0 \pm 0.1$. The lattice studies of refs. [49, 50] find results in complete agreement, however, with somewhat larger uncertainties. The analytic non-perturbative approaches of refs. [52, 53, 54, 55], employing either the large-N_c expansion or dispersion relations, on the other hand, find results that are generally larger than the lattice findings, although certainly compatible within the uncertainties. However, these analyses have been performed in the chiral limit, and as was demonstrated in [55], chiral corrections tend to decrease R_8. Therefore, in order to be more conservative, and in view of the fact that there could be additional effects of chiral logarithms when relating the matrix elements computed on the lattice to the physically relevant ones, in what follows we shall use:

$$R_8 = 1.0 \pm 0.2\,, \qquad \langle Q_8\rangle_2^{\mathrm{NDR}}(m_c) = (0.95 \pm 0.19)\,\mathrm{GeV}^3\,. \tag{3.3}$$

3.3 Determining R_6

The situation is less clear concerning the QCD penguin contribution parametrised by R_6. Therefore, we postpone a discussion of the present status of R_6, and we shall take a different approach. Putting together all the information presented thus far, let us investigate what can be said about our expectations for R_6. A summary of this analysis is presented in table 2. The second column shows central values for R_6 corresponding to the four inputs for R_8 as given in the first column. The remaining columns display the variation of R_6 while varying the dominant input parameters, and the last column contains the total uncertainty when all individual errors are added in quadrature.

A graphical representation of our results of table 2 in the spirit of the "ε'/ε-path" of ref. [56] (see also refs. [46, 9]) is given in figure 1. The solid straight line corresponds to the central values given in table 2, whereas the short-dashed lines are the uncertainties due to a variation of the input parameters. (Let us note that all our errors should be considered as 1σ deviations.) Next, the vertical long-dashed lines indicate the lattice range for R_8 [51], whereas the ellipse describes the correlation between R_6 and R_8 implied by the data on ε'/ε when taking into account the more conservative constraint on R_8 given in eq. (3.3). From the allowed region within the ellipse, we can infer our central result for R_6, which

R_8	R_6	ε'/ε	$\text{Im}\lambda_t$	$\Lambda_{\overline{\text{MS}}}$	Ω_{IB}	m_t	m_s	total
0.8	1.143	$^{+0.070}_{-0.070}$	$^{-0.051}_{+0.060}$	$^{-0.061}_{+0.065}$	$^{+0.092}_{-0.078}$	$^{+0.018}_{-0.017}$	$^{+0.007}_{-0.012}$	$^{+0.147}_{-0.133}$
1.0	1.222	$^{+0.070}_{-0.070}$	$^{-0.051}_{+0.060}$	$^{-0.061}_{+0.064}$	$^{+0.099}_{-0.084}$	$^{+0.024}_{-0.023}$	$^{+0.007}_{-0.012}$	$^{+0.152}_{-0.138}$
1.2	1.301	$^{+0.070}_{-0.070}$	$^{-0.051}_{+0.060}$	$^{-0.061}_{+0.064}$	$^{+0.107}_{-0.090}$	$^{+0.029}_{-0.028}$	$^{+0.007}_{-0.012}$	$^{+0.158}_{-0.142}$
2.2	1.695	$^{+0.070}_{-0.070}$	$^{-0.051}_{+0.060}$	$^{-0.059}_{+0.062}$	$^{+0.143}_{-0.121}$	$^{+0.058}_{-0.057}$	$^{+0.007}_{-0.012}$	$^{+0.190}_{-0.170}$

Table 2: R_6 as a function of R_8 for four relevant values. In addition, we give the variation of R_6 while varying all other input parameters. For a more detailed explanation see the text.

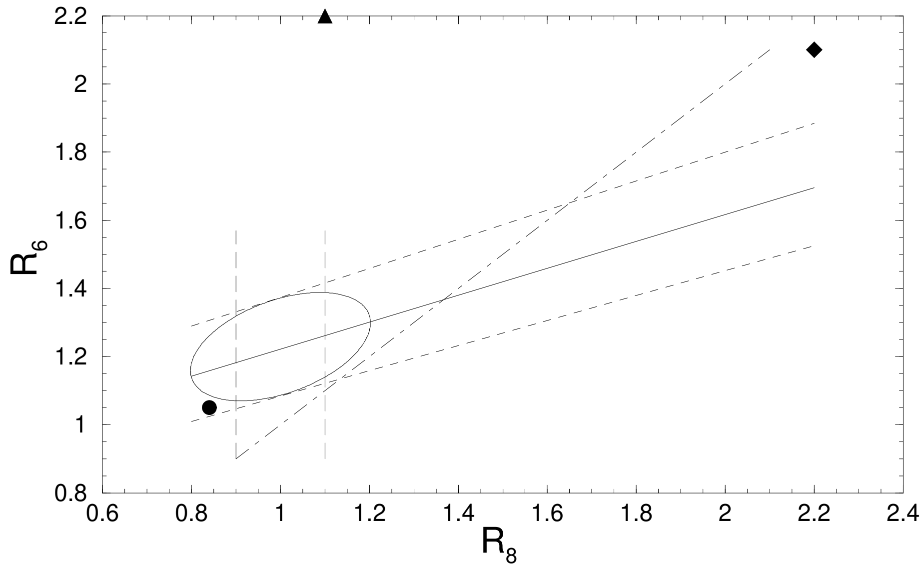

Figure 1: R_6 as a function of R_8. For a detailed explanation see the text.

reads:

$$R_6 = 1.23 \pm 0.16\,, \qquad \langle Q_6 \rangle_0^{\text{NDR}}(m_c) = -\,(0.73 \pm 0.10)\,\text{GeV}^3\,. \tag{3.4}$$

Finally, the full triangle, circle and diamond in figure 1 represent the central results of refs. [13, 54], [10] as well as [9] respectively, which will be discussed in more detail below, and the dashed-dotted line shows the strict large-N_c relation $R_6 = R_8$.

Comparing (3.4) with (3.3), we observe that if the matrix element of Q_8 is indeed given by (3.3) than the data on ε'/ε seem to require

$$\frac{R_6}{R_8} = 1.23 \pm 0.29\,, \qquad \langle Q_6 \rangle_0^{\text{NDR}}(m_c) = -\,(0.77 \pm 0.19)\,\langle Q_8 \rangle_2^{\text{NDR}}(m_c)\,. \tag{3.5}$$

Due to substantial uncertainties, these results are compatible with the large-N_c expectations in (2.12) but the strong chiral suppression of the matrix element $\langle Q_6 \rangle_0^{\text{NDR}}(m_c)$ with respect to $\langle Q_8 \rangle_2^{\text{NDR}}(m_c)$ found in the latter approach, although visible in (3.5), is not fully supported by the data. However, we would like to emphasise that this result is a consequence of (3.3). We will return to this point in the next section, where we shall compare the results in (3.3), (3.4) and (3.5) with other approaches.

	$R_6 = 1.0$		$R_6 = 1.2$		$R_6 = 1.4$	
$\Lambda^{(4)}_{\overline{\rm MS}}$ [MeV]	$R_8 = 0.8$	$R_8 = 1.0$	$R_8 = 0.8$	$R_8 = 1.0$	$R_8 = 0.8$	$R_8 = 1.0$
310	12.2	10.5	16.4	14.8	20.7	19.0
340	13.3	11.5	17.9	16.1	22.5	20.7
370	14.6	12.6	19.5	17.6	24.5	22.5

Table 3: The ratio ε'/ε in units of 10^{-4} for $m_t = 165\,\text{GeV}$ and various values of $\Lambda^{(4)}_{\overline{\rm MS}}$, R_6, R_8.

3.4 The dependence of ε'/ε on R_6 and R_8

In addition, it is also of interest to show the values of ε'/ε for specific values of R_6, R_8 and $\Lambda^{(4)}_{\overline{\rm MS}}$. We do this in table 3. The results in table 3 should be self explanatory. Depending on the values of the parameters $\Lambda^{(4)}_{\overline{\rm MS}}$, R_6 and R_8, one finds values of ε'/ε that are within or outside the one standard deviation experimental range given in (1.1). However, the remarkable feature of the values in this table is that most of them are within two standard deviations from the experimental value.

4. Comparison with other analyses

Let us now compare our findings of (3.3), (3.4) and (3.5) with other recent estimates of these quantities.

In the approach of refs. [15, 12, 10], the enhancement of the QCD penguin matrix element $\langle Q_6\rangle_0$ above the factorisation result was attributed to the strong final-state interactions in the final state pion system. In [15, 12], the final-state interactions were taken into account through dispersion relation techniques which resulted in an Omnés type exponential, giving an enhancement factor 1.55 ± 0.10 which provides full agreement with the experimental result for ε'/ε.[1] No such enhancement has been found in the case of $\langle Q_8\rangle_2$, for which a mild suppression factor 0.92 ± 0.03 was obtained [10]. Furthermore, the results of the Omnés approach were corroborated by an explicit computation within chiral perturbation theory at one loop.

However, the strange mass used in ref. [10] corresponds to $(m_s + m_d)(m_c) = 135\,\text{MeV}$, thus being 10% higher than the value advocated in the present work. Translating the findings of [10] for the case of Omnés resummed final-state interactions into R_6 and R_8, we obtain

$$R_6 = 1.05 \pm 0.06\,, \qquad R_8 = 0.84 \pm 0.05\,, \tag{4.1}$$

in reasonable agreement with our values, albeit pointing to a slightly different scenario as to their composition into B-parameter and strange mass. With the present uncertainty in the strange quark mass, which is not included in the errors given in eq. (4.1), we are, however, unable to decide which is the correct scenario. The central result of (4.1) is also indicated as the full circle in figure 1.

In the very recent article [9], the matrix element of the QCD penguin operator Q_6 was calculated within an approach which was able to take into account the dominant $\mathcal{O}(n_f/N_c)$

[1] Further discussion of this approach can also be found in refs. [57, 58].

corrections to the large-N_c result, where n_f is the number of active quark flavours. In addition, the authors demonstrated an explicit cancellation of the renormalisation scale dependence between Wilson coefficients and matrix elements. Numerically, in our language, the results of [9] and [53] would correspond to

$$R_6 = 2.1 \pm 1.1\,, \qquad R_8 = 2.2 \pm 0.4\,. \tag{4.2}$$

As seen in figure 1, the central result, displayed as the full diamond, is roughly consistent with the experimental value for ε'/ε, but lies far outside the ellipse found here. However, in the approach of [9], the dominant uncertainty for R_6 is due to the value of the quark condensate, and, as already remarked above, both parameters are presently only calculated in the chiral limit. Thus, in our opinion, at the moment it is premature to draw any definite conclusions from the results of [53] and [9].

Similar results of a rather large value of the matrix element $\langle Q_6\rangle_0$ were also found earlier in the framework of the $1/N_c$ expansion at NLO with chiral perturbation theory at the leading order [13, 11]. With the value $m_s(2\,\mathrm{GeV}) = 105\,\mathrm{MeV}$ advocated there, the finding $B_6^{(1/2)} = 2.5 \pm 0.4$ leads to $R_6 = 2.2 \pm 0.4$, in perfect agreement with the result of eq. (4.2). Together with the result $R_8 = 1.1 \pm 0.3$ [54], in figure 1 the findings of [13, 11, 54] are displayed as the full triangle. It should be clear that this set of parameters leads to a value of ε'/ε much larger than the experimental average. Nevertheless, one should note that like in ref. [9], also in [13, 11], the penguin matrix element $\langle Q_6\rangle_0$ was expressed in terms of the quark condensate. It is, however, well known that the chiral corrections to the GMOR relation which relates the parametrisations (2.10), and the one in terms of the quark condensate, in the strange quark case are large, of order 50% [48]. This can easily lead to a factor of two difference for both parametrisations, and one has to wait for a complete computation at the next-to-leading order, before it is clear which one is closer to the true result.

To conclude this section, we further elaborate on the strict large-N_c limit and perform the following exercise. We set

$$B_6^{(1/2)} = B_8^{(3/2)} = 1.0 \quad [27,\,59] \qquad \text{and} \qquad \hat{B}_K = 0.75 \quad [60,\,61], \tag{4.3}$$

In that case one finds [35]

$$\mathrm{Im}\lambda_t = (1.34 \pm 0.06)\cdot 10^{-4}\,. \tag{4.4}$$

Setting $\mathrm{Im}\lambda_t$ to this central value, in table 4 we show the results for ε'/ε as a function of $m_s(m_c)$ and $\Lambda^{(4)}_{\overline{\mathrm{MS}}}$.

We observe that for $m_s(m_c) \approx 105\,\mathrm{MeV}$ or equivalently $m_s(2\,\mathrm{GeV}) \approx 90\,\mathrm{MeV}$ and $\Lambda^{(4)}_{\overline{\mathrm{MS}}} \approx 340\,\mathrm{MeV}$, the results in table 4 are fully consistent with the data on ε'/ε. The required value of m_s is smaller, but consistent with (3.2) and in the ballpark of values found in the most recent lattice simulations with dynamical fermions [44, 62, 63]. As can be seen from the dashed-dotted line in figure 1, the required value for the large-N_c condition $R_6 = R_8$ is

$$R_6 = R_8 = 1.36 \pm 0.3\,, \tag{4.5}$$

which is close to the ellipse found here but outside of it.

JHEP01(2004)048

$\Lambda^{(4)}_{\overline{\rm MS}}$ [MeV]	$m_s(m_c) = 115\,\mathrm{MeV}$	$m_s(m_c) = 105\,\mathrm{MeV}$	$m_s(m_c) = 95\,\mathrm{MeV}$
310	10.8	13.4	16.8
340	11.8	14.6	18.2
370	12.9	15.9	19.9

Table 4: The ratio ε'/ε in units of 10^{-4} for the strict large-N_c results $B_6^{(1/2)} = B_8^{(3/2)} = 1.0$, $\hat{B}_K = 0.75$ and various values of $\Lambda^{(4)}_{\overline{\rm MS}}$ and $m_s(m_c)$.

5. Conclusions

In this letter, we have reanalysed the ratio ε'/ε in view of the improved experimental data and an improved information on the relevant theoretical input parameters. After the uncertainties on $\mathrm{Im}\lambda_t$ have been reduced significantly during the last four years and some consensus has been reached on the matrix element of the dominant electroweak penguin operator Q_8, the main uncertainty in the estimate of ε'/ε in the SM is the matrix element of the leading QCD penguin operator Q_6. Assuming that ε'/ε is fully dominated by the SM contributions, we have determined the ratio R_6 and the matrix element $\langle Q_6\rangle_0^{\mathrm{NDR}}(m_c)$ from the data on ε'/ε with the results given in (3.4) and (3.5).

We have also shown that with $\hat{B}_K = 0.75$ and $m_s(m_c) \approx 105\,\mathrm{MeV}$, the large-$N_c$ values of $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ with their Wilson coefficients calculated within the SM are consistent with the experimental data on ε'/ε. This should be contrasted with the situation of 1999, when the large-N_c estimates of ε'/ε were by a factor of two to three lower than the data. The reasons for this change have already been listed in section 1.

Finally, we have shown in figure 1 the correlation between R_6 and R_8 as implied by the experimental data on ε'/ε.

There are three messages from our paper:

- If indeed $R_8 = 1.0 \pm 0.2$ as indicated by several recent estimates, then the data on ε'/ε imply
$$R_6 = 1.23 \pm 0.16\,, \qquad \frac{R_6}{R_8} = 1.23 \pm 0.29\,, \qquad \langle Q_6\rangle_0^{\mathrm{NDR}}(m_c) = -(0.77 \pm 0.19)\langle Q_8\rangle_2^{\mathrm{NDR}}(m_c)\,. \tag{5.1}$$
This is in accordance with the results in [15, 12, 10], but as seen in figure 1 it differs from the large-N_c approaches in [27, 28, 29], [13, 11, 54] and [9] with the first one closest to our findings.

- The large-N_c approach of [27, 28, 29] can only be made consistent with data provided
$$R_6 = R_8 = 1.36 \pm 0.3 \tag{5.2}$$
and $\langle Q_8\rangle_2^{\mathrm{NDR}}(m_c)$ is larger than obtained by most recent approaches discussed above. This would require $m_s(m_c) \approx 105\,\mathrm{MeV}$ which is still compatible with (3.2). Moreover, lower values of $m_s(m_c)$ are indicated by the recent lattice simulations with dynamical fermions [44, 62, 63], but the situation certainly requires further study.

JHEP01(2004)048

- Higher values of $\langle Q_6\rangle_0^{\rm NDR}(m_c)$ and $\langle Q_8\rangle_2^{\rm NDR}(m_c)$ are obtained in the approach of [9] but as the calculations have been done in the chiral limit, it would be premature to draw definite conclusions at present. From the point of view of the approach in [27, 28, 29] the values $R_6 \approx R_8 \approx 2$ would correspond to rather small values of $m_s(m_c)$ so that a better interpretation would be a significant departure of the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from unity. As the analysis [9] goes beyond the strict large-N_c limit, this is certainly conceivable.

As shown in figure 1, all these three scenarios are consistent with the data. Which of these pictures of ε'/ε is correct, can only be answered by calculating $\langle Q_6\rangle_0$ by means of non-perturbative methods that go beyond the large-N_c limit and are reliable. Moreover, the calculation of the strange quark mass m_s and of $\langle Q_8\rangle_2$ should be improved. Such calculations are independent of the assumption about the role of new physics in ε'/ε that we have made in order to extract $\langle Q_6\rangle_0$ from the data.

If the values for these matrix elements and $R_{6,8}$ will be found one day to lie significantly outside the allowed region in figure 1, new physics contributions to ε'/ε will be required in order to fit the experimental data.

Acknowledgments

M.J. would like to thank D. Becirevic, S. Peris and J. Prades for discussions. This research was partially supported by the German 'Bundesministerium für Bildung und Forschung' under contract 05HT1WOA3 and by the 'Deutsche Forschungsgemeinschaft' (DFG) under contract Bu.706/1-2. M.J. would also like to thank the Deutsche Forschungsgemeinschaft for support.

References

[1] NA48 collaboration, J.R. Batley et al., *A precision measurement of direct CP-violation in the decay of neutral kaons into two pions*, *Phys. Lett.* **B 544** (2002) 97 [hep-ex/0208009].

[2] NA48 collaboration, A. Lai et al., *A precise measurement of the direct CP-violation parameter* $\mathrm{Re}(\varepsilon'/\varepsilon)$, *Eur. Phys. J.* **C 22** (2001) 231 [hep-ex/0110019].

[3] KTeV collaboration, A. Alavi-Harati et al., *Measurements of direct CP-violation, CPT symmetry and other parameters in the neutral kaon system*, *Phys. Rev.* **D 67** (2003) 012005 [hep-ex/0208007].

[4] KTeV collaboration, A. Alavi-Harati et al., *Observation of direct CP-violation in* $K_{S,L} \to \pi\pi$ *decays*, *Phys. Rev. Lett.* **83** (1999) 22 [hep-ex/9905060].

[5] NA31 collaboration, H. Burkhardt et al., *First evidence for direct CP-violation*, *Phys. Lett.* **B 206** (1988) 169.

[6] NA31 collaboration, G.D. Barr et al., *A new measurement of direct CP-violation in the neutral kaon system*, *Phys. Lett.* **B 317** (1993) 233.

[7] E731 collaboration, L.K. Gibbons et al., *Measurement of the CP-violation parameter* $\mathrm{Re}(\varepsilon'/\varepsilon)$, *Phys. Rev. Lett.* **70** (1993) 1203.

[8] M. Holder, *CP and CPT violation in K^0 decays*, *Acta Phys. Polon.* **33** (2002) 4379.

[9] T. Hambye, S. Peris and E. de Rafael, *$\Delta I = 1/2$ and ε'/ε in large-N_c QCD*, *J. High Energy Phys.* **05** (2003) 027 [hep-ph/0305104].

[10] E. Pallante, A. Pich and I. Scimemi, *The Standard Model prediction for ε'/ε*, *Nucl. Phys.* **B 617** (2001) 441 [hep-ph/0105011].

[11] J. Bijnens and J. Prades, *Chiral limit prediction for $\varepsilon'_K/\varepsilon_K$ at NLO in $1/N_c$*, *Nucl. Phys.* **96** *(Proc. Suppl.)* (2001) 354 [hep-ph/0010008].

[12] E. Pallante and A. Pich, *Final state interactions in kaon decays*, *Nucl. Phys.* **B 592** (2001) 294 [hep-ph/0007208].

[13] J. Bijnens and J. Prades, *$\varepsilon'_K/\varepsilon_K$ in the chiral limit*, *J. High Energy Phys.* **06** (2000) 035 [hep-ph/0005189].

[14] S. Bertolini, J.O. Eeg and M. Fabbrichesi, *An updated analysis of ε'/ε in the Standard Model with hadronic matrix elements from the chiral quark model*, *Phys. Rev.* **D 63** (2001) 056009 [hep-ph/0002234].

[15] E. Pallante and A. Pich, *Strong enhancement of ε'/ε through final state interactions*, *Phys. Rev. Lett.* **84** (2000) 2568 [hep-ph/9911233].

[16] T. Hambye, G.O. Köhler, E.A. Paschos and P.H. Soldan, *Analysis of ε'/ε in the $1/N_c$ expansion*, *Nucl. Phys.* **B 564** (2000) 391 [hep-ph/9906434].

[17] S. Bosch, A.J. Buras, M. Gorbahn, S. Jäger, M. Jamin, M.E. Lautenbacher, and L. Silvestrini, *Standard Model confronting new results for ε'/ε*, *Nucl. Phys.* **B 565** (2000) 3 [hep-ph/9904408].

[18] T. Hambye, G.O. Köhler, E.A. Paschos, P.H. Soldan and W.A. Bardeen, *$1/N_c$ corrections to the hadronic matrix elements of Q_6 and Q_8 in $K \to \pi\pi$ decays*, *Phys. Rev.* **D 58** (1998) 014017 [hep-ph/9802300].

[19] A.J. Buras, M. Jamin and M.E. Lautenbacher, *The anatomy of ε'/ε beyond leading logarithms with improved hadronic matrix elements*, *Nucl. Phys.* **B 408** (1993) 209 [hep-ph/9303284].

[20] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *ε'/ε at the next-to-leading order in QCD and QED*, *Phys. Lett.* **B 301** (1993) 263 [hep-ph/9212203].

[21] S. Bertolini, M. Fabbrichesi and J.O. Eeg, *Theory of the CP-violating parameter ε'/ε*, *Rev. Mod. Phys.* **72** (2000) 65 [hep-ph/9802405].

[22] A.J. Buras, *Weak hamiltonian, CP-violation and rare decays*, hep-ph/9806471, lectures presented in *Probing the Standard Model of particle interactions* Les Houches, 1997.

[23] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Effective hamiltonians for $\Delta S = 1$ and $\Delta B = 1$ nonleptonic decays beyond the leading logarithmic approximation*, *Nucl. Phys.* **B 370** (1992) 69, addendum *ibid.* **B375** (1992) 501.

[24] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Two loop anomalous dimension matrix for $\Delta S = 1$ weak nonleptonic decays, 1. $\mathcal{O}(\alpha_s^2)$*, *Nucl. Phys.* **B 400** (1993) 37 [hep-ph/9211304].

[25] A.J. Buras, M. Jamin and M.E. Lautenbacher, *Two loop anomalous dimension matrix for $\Delta S = 1$ weak nonleptonic decays, 2. $\mathcal{O}(\alpha_e \alpha_s)$*, *Nucl. Phys.* **B 400** (1993) 75 [hep-ph/9211321].

[26] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *The $\Delta S = 1$ effective hamiltonian including next-to-leading order QCD and QED corrections*, *Nucl. Phys.* **B 415** (1994) 403 [hep-ph/9304257].

[27] W.A. Bardeen, A.J. Buras and J.M. Gerard, *The $\Delta I = 1/2$ rule in the large-N limit*, *Phys. Lett.* **B 180** (1986) 133.

[28] W.A. Bardeen, A.J. Buras and J.M. Gerard, *The $K \to \pi\pi$ decays in the large-N limit: quark evolution*, *Nucl. Phys.* **B 293** (1987) 787.

[29] W.A. Bardeen, A.J. Buras and J.M. Gerard, *A consistent analysis of the $\Delta I = 1/2$ rule for K decays*, *Phys. Lett.* **B 192** (1987) 138.

[30] A.J. Buras, P. Gambino, M. Gorbahn, S. Jäger and L. Silvestrini, *ε'/ε and rare K and B decays in the MSSM*, *Nucl. Phys.* **B 592** (2001) 55 [hep-ph/0007313].

[31] G. Buchalla, A.J. Buras and M.K. Harlander, *The anatomy of ε'/ε in the Standard Model*, *Nucl. Phys.* **B 337** (1990) 313.

[32] Particle Data Group collaboration, K. Hagiwara et al., *Review of particle physics*, *Phys. Rev.* **D 66** (2002) 010001.

[33] Y.-Y. Keum, U. Nierste and A.I. Sanda, *A short look at ε'/ε*, *Phys. Lett.* **B 457** (1999) 157 [hep-ph/9903230].

[34] M. Battaglia et al., *The CKM matrix and the unitarity triangle*, proceedings of the workshop, Geneva, Switzerland, february 13-16, 2002, hep-ph/0304132.

[35] A. Stocchi, *private communication.*

[36] V. Cirigliano, A. Pich, G. Ecker and H. Neufeld, *Isospin violation in ε'*, *Phys. Rev. Lett.* **91** (2003) 162001 [hep-ph/0307030].

[37] G. Ecker, G. Müller, H. Neufeld and A. Pich, *$\pi^0\eta$ mixing and CP-violation*, *Phys. Lett.* **B 477** (2000) 88 [hep-ph/9912264].

[38] C.E. Wolfe and K. Maltman, *The strong isospin breaking correction for the gluonic penguin contribution to ε'/ε at next-to-leading order in the chiral expansion*, *Phys. Rev.* **D 63** (2001) 014008 [hep-ph/0007319].

[39] A.J. Buras and M.E. Lautenbacher, *An analytic formula and an upper bound for ε'/ε in the Standard Model*, *Phys. Lett.* **B 318** (1993) 212 [hep-ph/9309203].

[40] E. Gámiz, M. Jamin, A. Pich, J. Prades and F. Schwab, *Determination of m_s and $|V_{us}|$ from hadronic τ decays*, *J. High Energy Phys.* **01** (2003) 060 [hep-ph/0212230].

[41] S. Chen, M. Davier, E. Gámiz, A. Höcker, A. Pich, and J. Prades, *Strange quark mass from the invariant mass distribution of Cabibbo-suppressed τ decays*, *Eur. Phys. J.* **C 22** (2001) 31 [hep-ph/0105253].

[42] M. Jamin, J.A. Oller and A. Pich, *Light quark masses from scalar sum rules*, *Eur. Phys. J.* **C 24** (2002) 237 [hep-ph/0110194].

[43] K. Maltman and J. Kambor, *Decay constants, light quark masses and quark mass bounds from light quark pseudoscalar sum rules*, *Phys. Rev.* **D 65** (2002) 074013 [hep-ph/0108227].

[44] H. Wittig, *Chiral effective lagrangian and quark masses*, *Nucl. Phys.* **119** *(Proc. Suppl.)* (2003) 59 [hep-lat/0210025].

[45] M. Ciuchini and G. Martinelli, *Theoretical status of ε'/ε*, *Nucl. Phys.* **99B** *(Proc. Suppl.)* (2001) 27 [hep-ph/0006056].

[46] J.F. Donoghue, *ε'/ε in the Standard Model*, in proceedings of *Kaon 2001*, F. Costantini, G. Isidori and M. Sozzi eds., p. 93.

[47] M. Gell-Mann, R.J. Oakes, and B. Renner, *Behavior of current divergences under* $SU(3) \times SU(3)$, *Phys. Rev.* **175** (1968) 2195.

[48] M. Jamin, *Flavour-symmetry breaking of the quark condensate and chiral corrections to the Gell-Mann–Oakes–Renner relation*, *Phys. Lett.* **B 538** (2002) 71 [hep-ph/0201174].

[49] RBC collaboration, T. Blum et al., *Kaon matrix elements and CP-violation from quenched lattice QCD, I. The 3-flavor case*, *Phys. Rev.* **D 68** (2003) 114506 [hep-lat/0110075].

[50] CP-PACS collaboration, S. Aoki et al., *Calculation of $K \to \pi\pi$ decay amplitudes from $K \to \pi$ matrix elements in quenched domain-wall QCD*, *Nucl. Phys.* **106** *(Proc. Suppl.)* (2002) 332 [hep-lat/0110142].

[51] SPQCDR collaboration, D. Becirevic et al., *Kaon weak matrix elements with Wilson fermions*, *Nucl. Phys.* **119** *(Proc. Suppl.)* (2003) 359 [hep-lat/0209136].

[52] S. Narison, *New QCD-estimate of the kaon penguin matrix elements and ε'/ε*, *Nucl. Phys.* **B 593** (2001) 3 [hep-ph/0004247].

[53] M. Knecht, S. Peris and E. de Rafael, *A critical reassessment of Q_7 and Q_8 matrix elements*, *Phys. Lett.* **B 508** (2001) 117 [hep-ph/0102017].

[54] J. Bijnens, E. Gamiz and J. Prades, *Matching the electroweak penguins Q_7, Q_8 and spectral correlators*, *J. High Energy Phys.* **10** (2001) 009 [hep-ph/0108240].

[55] V. Cirigliano, J.F. Donoghue, E. Golowich and K. Maltman, *Improved determination of the electroweak penguin contribution to ε'/ε in the chiral limit*, *Phys. Lett.* **B 555** (2003) 71 [hep-ph/0211420].

[56] A.J. Buras and J.-M. Gerard, *What is the $(\varepsilon'/\varepsilon)_{\rm exp}$ telling us?*, *Phys. Lett.* **B 517** (2001) 129 [hep-ph/0106104].

[57] A.J. Buras et al., *Final state interactions and ε'/ε: a critical look*, *Phys. Lett.* **B 480** (2000) 80 [hep-ph/0002116].

[58] M. Büchler, G. Colangelo, J. Kambor and F. Orellana, *A note on the dispersive treatment of $K \to \pi\pi$ with the kaon off-shell*, *Phys. Lett.* **B 521** (2001) 29 [hep-ph/0102289].

[59] A.J. Buras and J.M. Gérard, *Isospin breaking contributions to ε'/ε*, *Phys. Lett.* **B 192** (1987) 156.

[60] B.D. Gaiser, T. Tsao and M.B. Wise, *Parameters of the six quark model*, *Ann. Phys. (NY)* **132** (1981) 66.

[61] A.J. Buras and J.M. Gérard, *$1/N$ expansion for kaons*, *Nucl. Phys.* **B 264** (1986) 371.

[62] HPQCD collaboration, J. Hein, C. Davies, G.P. Lepage, Q. Mason and H. Trottier, *On the strange quark mass with improved staggered quarks*, *Nucl. Phys.* **119** *(Proc. Suppl.)* (2003) 317 [hep-lat/0209077].

[63] R. Gupta, *Quark masses from the lattice*, talk presented at *CKM matrix and unitarity triangle*, Durham, March 2003.

JHEP01(2004)048